Periodic Vehicle Routing Problem in a Health Unit

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Abstract: In logistics of home health care services in the Health Units, the managers and nurses need to carry out the schedule and the vehicles routes for the provision of care at the patients' homes. Currently, in Portugal, these services are increasingly used but the problem is still, usually, solved manually and without computational resources. The increased demand for home health care due to the boost of the elderly people number entails a high associated cost which, sometimes, does not guarantee the quality of the service. In this sense, the periodic vehicle routing problem is a generalization of the classical vehicle routing problem in which routes are determined for a time horizon of several days. In this work, it is provided a periodic vehicle routing problem applied in the Health Unit in Bragança. An integer linear programming formulation for the real database, allowed to solve the problem in an efficient and optimized way using the CPLEX® software.

1 INTRODUCTION

Home Health Care (HHC) is a growing medical service in many countries (Benzarti et al., 2013). One of the tasks of the HHC involves the management decisions, that is, it is necessary to create the planning and the routing of vehicles or nurses/doctors in the domiciliary services. In Portugal, there is a large number of elderly people who need support in their homes, therefore the operational management problems related to home care visits are very important.

The HHC service in a health care unit has a fleet of vehicles used by nurses/doctors to travel to the patients locations. A periodic aspect is also involved, since the jobs need to be performed, repeatedly, during a specific visits schedule. Usually, the HHC management is solved manually by a senior nurse or manager, who often spends a huge amount of time to create next week's schedule (Fikar and Hirsch, 2017). However, the scheduling and routing problems in this area are complex procedures (Nickel et al., 2012) due to the mathematical modeling complexity and, more importantly, the absence of computational resources in the Health Unit. From a mathematical point of view, this problem has special interest, because it is an NP-hard problem, such as periodic vehicle routing problem (PVRP). In this sense, the PVRP is a generalization of the classical vehicle routing problem (VRP) where routes are determined for a planning horizon of multiple periods with some customers demanding multiple visits (Angelelli and Speranza, 2002; Beltrami and Bodin, 1974; Cordeau et al., 1997). This approach is different from route planning, in the sense that route planning performs the scheduling for one day and this approach performs for a certain period of time (Campbell and Wilson, 2014). Many real-world applications in transportation systems require finding, for a fleet of vehicles, a set of minimum cost routes that satisfy orders or services requested by customers/patients, over a given planning horizon (e.g. one week). This problem is very important in real-world applications such as the distribution for bakery companies (Pacheco et al., 2012) and also in blood product distribution (Hemmelmayr et al., 2009), among other services. Some authors address and describe the HHC problem through a hybridization of constraint programming and meta-heuristics including simulated annealing and tabu search (Bertels and Fahle, 2006) or genetic algorithm (Alves et al., 2018). Other approaches in the HHC problem can be found in Trautsamwieser et al. (Trautsamwieser et al., 2011) and Nickel et al. (Nickel et al., 2012). Thus and although there is already an extensive operational research in HHC, many focus only on the daily scheduling of the problem of home visits. However, in these types of health services, the scheduling and routing may be periodic in order to group periods of home visits and to respect regularities imposed by patients (becoming a PVRP). Typically, the PVRP has a time horizon of T days, and there is a demand and regularity of delivery for each client or patient indicating how many times within the period of T days the customer should be visited. The solution to the PVRP is to find the T sets of routes that together satisfy the constraints of the requested quantity and regularity and also minimize travel distances.

The applicability and versatility of the problem has led to extensive research addressing both new applications and solution methods. It has become a topic with extensive studies in operational research with extension in diverse services. However, only limited research has been conducted into the HHC problem.

The paper is organized as follows: the next section, briefly reviews the main definitions and the problem approach. Section 3 presents the integer linear programming model for the PVRP. The case study, based on real data from a Bragança Health Unit, is described in Section 4. In Section 5 the results are analyzed and discussed. Finally, some conclusions and future work are drawn in Section 6.

2 PVRP DEFINITION

The problem addressed in this article is related to the schedule and routes of home care visits outside the Health Unit of Bragança in a certain time horizon.

The home care service can be provided by nursing teams of a Health Unit, which involves the scheduling of nurses and the definition of transportation routes to the patients' homes. Currently, home visits are planned manually and without computational support, which can lead to not obtaining the optimal solution, especially when certain constraints must be met (distances and costs involved, time windows, among others). Thus, it is necessary to develop approaches that are able to overcome the difficulties that may occur in these services, such as the accumulation of delays on routes, the definition of routes that are too long, the planning without feasible options for replacing nurses, and the difficulties in managing periodic visits.

Therefore, the first approach is to define the general characteristics of the problem, such as the number and characterization of health professionals, the number of available vehicles (m), the number of patients (n) and treatments they need, locations that can be traveled and their distances, among others. These data allow us to formulate and model the problem, in an attempt to minimize the time spent on visits, reduce costs and provide support for decision making.

In this paper, an approach for the optimization of the PVRP, which deals with home care visits in order to perform treatments for patients belonging to a Health Unit is presented. For this problem the number of vehicles involved in the home care and the patients requesting this type of health services is considered.

The problem in establishing periodic routes produces a schedule within a time horizon of T days, where each patient i, in addition to the number of visits, may have a minimum and/or maximum time period between successive visits, e_i, i.e. the number of visits combined with the time period, leads to the set of patterns p_i that corresponds to the possible patterns of visits in the time horizon T. For example, if the planning is executed for a time horizon of 5 days ($T = 5$), with a time period between visits of at least two days ($e_i \geq 2$), the set of patterns will be $p_i = \{\{1,4\},\{1,5\},\{2,5\}\}$. This corresponds to visiting the patient twice during the 5 days of the planning period and should be done on days 1 and 4, or on days 1 and 5 or alternatively on days 2 and 5, and no other pattern of visits is possible. The problem is to simultaneously select the pattern of each patient and establish the routes for each day of the time horizon.

The Figure 1 shows an example of PVRP in which routes 1 and 2 will have to occur at different times (because they have points in common).

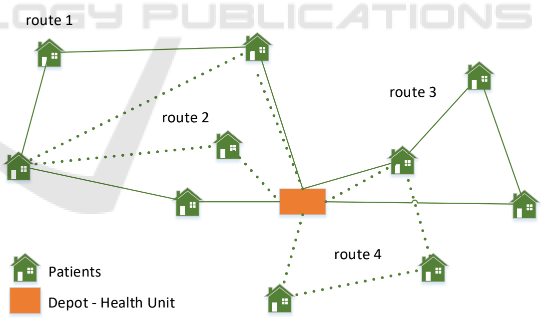

Figure 1: Illustration of the PVRP.

Similarly, routes 3 and 4 will also have to occur in different periods. In the PVRP, the cost of visiting patients is independent of the vehicles that carry it, not considering restrictions that include incompatibilities between vehicles and patients. The matrix $cost = (c_{ij})$, with rank n, reflects the time spent on the route between patient i and patient j.

The PVRP consists of establishing r routes of lower cost in order to satisfy the time horizon and certain assumptions, such that:

- Each patient belongs to exactly one route.

- Each route starts and ends at the depot (Health Unit of Bragança)

3 FORMULATION OF THE INTEGER LINEAR PROGRAMMING MODEL

The PVRP can be considered as an integer linear optimization problem where it is not only established the scheduling of patients visits, but also the design of the routes for the different days of the time horizon in order to minimize the costs associated with all the routes of the time period under study. Consider the following input:

- $L = \{1, \ldots, T\}$ is the set of days in the planning horizon;
- $N = \{1, \ldots, n\}$ is the set of patients;
- The index of a route starting or ending at the depot has the value $\{0\}$;
- $M = \{1, \ldots, m\}$ is the set of vehicles;
- p_i is the set of patterns of patient i;
- u_i^{kl} is an integer corresponding to the place in the sequence of visits of vehicle k on day l for the patient i;
- a_i^{sl} is a variable to represent the s pattern that includes the patient i on day l.

The optimization problem is to find the optimal values for the binary decision variables such as x_{ij}^{kl} and y_i^s. Thus, the formulation of PVRP is defined as follows, where the binary variables take the form:

$$y_i^s = \begin{cases} 1, & \text{if patient } i \text{ is visited according to} \\ & \text{the } s \text{ pattern belonging to } p_i; \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ij}^{kl} = \begin{cases} 1, & \text{if the vehicle } k \text{ visits patient } j \\ & \text{immediately after patient } i \text{ on day } l; \\ 0, & \text{otherwise.} \end{cases}$$

The integer programming model is given by:

$$minimize \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} \left(\sum_{k=1}^{m} \sum_{l=1}^{t} x_{ij}^{kl} \right) \tag{1}$$

subject to:

$$\sum_{s \in p_i} y_i^s = 1, \quad \forall i \in N \tag{2}$$

$$\sum_{j=0}^{n} \sum_{k=1}^{m} x_{ij}^{kl} - \sum_{s \in p_i} a_i^{sl} y_i^s = 0, \quad \forall i \in N; \forall l \in L; i \neq j \tag{3}$$

$$\sum_{j=0}^{n} x_{ji}^{kl} - \sum_{j=0}^{n} x_{ij}^{kl} = 0, \quad \forall i \in N; \forall k \in M; \forall l \in L; i \neq j \tag{4}$$

$$\sum_{j=1}^{n} x_{0j}^{kl} \leq 1, \quad \forall k \in M; \forall l \in L \tag{5}$$

$$u_j^{kl} \geq u_i^{kl} - C(1 - x_{ij}^{kl}) + 1, \quad \forall i, j \in N; \forall k \in M; \forall l \in L; i \neq j \tag{6}$$

$$x_{ij}^{kl} \in \{0, 1\}, \quad \forall i, j \in N; \forall k \in M; \forall l \in L \tag{7}$$

$$y_i^s \in \{0, 1\}, \quad \forall i \in N; \forall s \in p_i \tag{8}$$

The objective function (1) represents the minimization of total cost, in this case the distance.

Constraints (2) ensure that each client will be assigned to one of the admissible visit patterns, while constraints (3) ensure that each client is visited exactly on the days of the pattern that is assigned. In turn, constraints (4) guarantee continuity of the route, i.e, a vehicle entering a vertex will have to leave it. Constraints (5) ensure that the number of available vehicles is not exceeded. Finally, the sub-circuit elimination restrictions can be expressed by constraints (6). In the presented formulation, the cost of visiting the patients is independent of the vehicles that do it, not considering restrictions that include incompatibilities between vehicles and patients. Thus, in the following section will be presented the real case study based on the Health Unit in Bragança, where the proposed formulation will be applied.

4 CASE STUDY

In this section a real case study of a Bragança Health Unit for a time horizon of $T = 5$ days (one week of home visits) is presented. In that same week, the Bragança Health Unit has five vehicles available for home care visits ($m = 5$), in which they visit fifteen patients ($n = 15$) with the need for treatments, where they are dispersed by their locations. Table 1 summarizes the data for the problem in question.

Table 1: Problem Data Information.

Vehicles	Patients	Locations	Time Horizon
5	15	15	5 days

According to the problem database, there are 5 vehicles available for home visits at the Health Unit of Bragança, which are homogeneous in terms of capacities and/or visits to patients. The number of patients

Table 2: Distances between patient locations.

	HU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
HU	0,0	15,2	18,0	22,4	25,0	20,6	11,2	21,2	26,2	32,0	25,5	33,5	15,0	11,2	32,0	30,4
1	15,2	0,0	32,6	14,6	32,2	32,2	24,8	21,0	31,6	17,8	15,6	26,4	16,6	26,4	46,9	45,4
2	18,0	32,6	0,0	34,4	20,2	23,9	16,4	36,2	36,1	47,4	43,3	50,3	23,4	9,4	21,2	13,0
3	22,4	14,6	34,4	0,0	25,0	42,7	33,5	35,4	45,0	15,0	29,2	40,3	11,2	32,0	53,2	47,2
4	25,0	32,2	20,2	25,0	0,0	41,2	31,6	46,1	50,5	40,0	47,2	57,0	15,8	25,5	41,2	29,2
5	20,6	32,2	23,9	42,7	41,2	0,0	10,0	20,6	13,9	50,0	33,5	35,4	35,4	15,8	20,0	29,2
6	11,2	24,8	16,4	33,5	31,6	10,0	0,0	20,6	19,8	42,4	30,4	35,4	25,5	7,1	22,4	25,5
7	21,2	21,0	36,2	35,4	46,1	20,6	20,6	0,0	12,2	36,4	14,1	15,0	33,5	26,9	40,3	46,1
8	26,2	31,6	36,1	45,0	50,5	13,9	19,8	12,2	0,0	48,1	26,2	24,2	40,8	26,9	33,4	42,9
9	32,0	17,8	47,4	15,0	40,0	50,0	42,4	36,4	48,1	0,0	25,0	35,4	25,5	43,0	64,0	60,4
10	25,5	15,6	43,3	29,2	47,2	33,5	30,4	14,1	26,2	25,0	0,0	11,2	32,0	35,0	52,2	55,0
11	33,5	26,4	50,3	40,3	57,0	35,4	35,4	15,0	24,2	35,4	11,2	0,0	42,4	41,2	55,2	60,8
12	15,0	16,6	23,4	11,2	15,8	35,4	25,5	33,5	40,8	25,5	32,0	42,4	0,0	22,4	43,0	36,1
13	11,2	26,4	9,4	32,0	25,5	15,8	7,1	26,9	26,9	43,0	35,0	41,2	22,4	0,0	21,2	20,0
14	32,0	46,9	21,2	53,2	41,2	20,0	22,4	40,3	33,4	64,0	52,2	55,2	43,0	21,2	0,0	15,8
15	30,4	45,4	13,0	47,2	29,2	29,2	25,5	46,1	42,9	60,4	55,0	60,8	36,1	20,0	15,8	0,0

is 15 that require and need treatments in their respective locations. Regarding the locations, it is necessary to know the different locations/cities of each patient belonging to the Health Unit and the respective temporal distance (minutes) between each one of them.

In this way, Table 2 presents the distances (in kilometers) between locations. The patients seek and need home visits with a certain regularity in the period of the visits ($T = 5$ days). Thus, Table 3 shows the number of times each patient should be visited.

Table 3: Regularity of visits required by each patient in the time horizon.

Period of visits they require for $T = 5$	
Patient 1	1
Patient 2	1
Patient 3	2
Patient 4	3
Patient 5	1
Patient 6	2
Patient 7	1
Patient 8	1
Patient 9	2
Patient 10	3
Patient 11	1
Patient 12	1
Patient 13	1
Patient 14	2
Patient 15	1

According to these data, it is also possible to identify some patterns about the number of visits required by each patient for the T period, knowing in advance that between two or more visits a day of interval is required. Thus, in this way it is possible to illustrate the different patterns according to Table 4:

Based on all the data, the main objective is to obtain vehicle routing/scheduling, finding the T sets of routes that satisfy the constraints and minimizing the total time required to carry out the trips, treatments and return to the starting point (Depot - Health Unit).

Table 4: Patterns of visits according to the period T.

Possible Pattern for visits to T = 5 days	
1 Visit	1, 2, 3, 4 or 5
2 Visit	1-3, 1-4, 1-5, 2-4, 2-5, 3-5
3 Visit	1-3-5

5 ANALYSIS AND DISCUSSION OF RESULTS

In this section the computational results of the model developed and proposed for the resolution of PVRP will be presented and analyzed.

The model was coded and implemented in the IBM® ILOG® CPLEX® Optimization Studio that supports Optimization Programming Language (OPL). The data of the real case under study was implemented according to the periodic home care visits approach and the results were obtained on an Intel (R) Core i7 CPU 2.2GHz PC with 6.0 GB of RAM.

The CPLEX® took about 11 hours to reach the solution. The obtained solution had the objective value of 473 and besides the regularity and periodicity of visits imposed by the patients in the time horizon, the model established the route patterns according to the objective reached. This solution indicates the minimum distance to be traveled (cost) for the vehicles to make the home visits routes, according to the defined time horizon and the regularity needed by the patients and nurses of the Health Unit of Bragança.

From Table 5, it is possible to get some statistical details, such as the number of variables used, the average value of the target solution, among other pa-

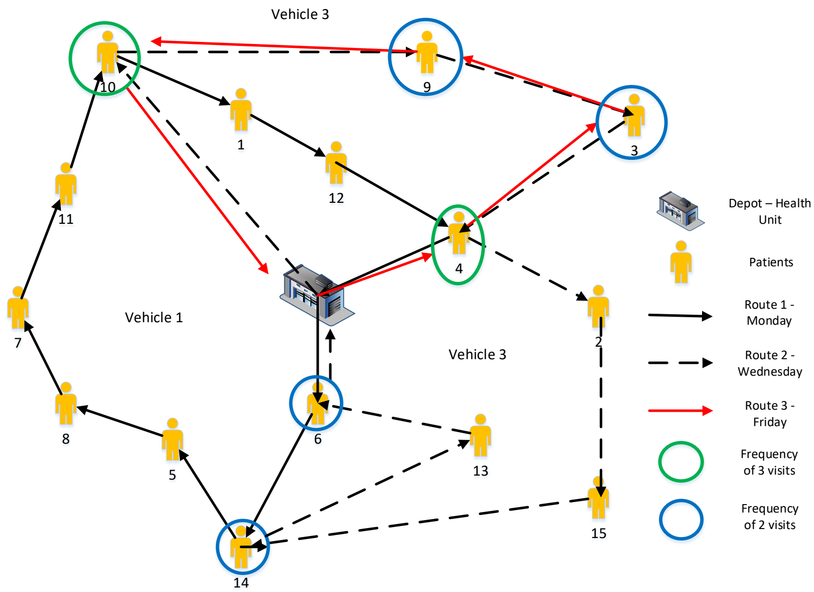

Figure 2: Representation of the PVRP solution.

rameters that allow to summarize the objective solution.

Table 5: Statistical values obtained by CPLEX®.

Statistic	Value
Constraints	5874
Variables	6955
Objective	473
Nodes	4724907
Mean Objective	547

Figure 2, in turn, illustrates the solution obtained, which includes the scheduling and the set of routes for the periodicity of home visits in question.

In this sense, two of the five vehicles available were used, where only one was used on two different days. Another thing to mention, is that from the defined time horizon only 3 days were subjected to the service of home care visits. These days allow to comply with all requirements according to the periodicity imposed. Certain patients require a higher frequency of visits, however it is possible to check those that belong to more than one route and day of visits, as can be seen in Figure 2. It is possible to conclude with the solution provided, that all patients are visited and all routes meet the time period between successive visits leading to admissible patterns.

The analysis showed that the problem can be solved by CPLEX® and provided the optimal solution of the single problem.

6 CONCLUSIONS AND FUTURE WORK

The PVRP are an extension of the classic vehicle routing problems. They are more difficult to solve when compared to the problems of classic routes, since they do not only aim to find a set of routes, but also the definition of a schedule of visits of the patients that minimizes the operational costs of the system/service within a time horizon. Since the route problem addressed is a periodic approach that covers a time horizon with several days, an integer linear programming model was developed using OPL/CPLEX® implementation for its resolution and optimization.

The developed model allows to solve problems of routes with time period between successive visits, meeting the demand and regularity of visits by the patients. In this way it was possible to establish the home visits in a Health Unit in Bragança, optimizing the routes in the service. In this work, a small real case of 15 patients is solved, allowing the Health Unit to plan routes and visits using computational support. However, replicating the problem with larger instances and/or increasing difficulty, may not be feasible due to the rather long computational time required.

The developed approach allows the planning of a set of routes that, with the existing resources, and guarantees the visit to all the locations with the time period defined by the managers, without incurring additional costs or deficiencies in the service. In this way, obtaining the solutions allows reducing and optimizing costs and routes, improving the health service provided and serving as a decision support system, which does not exist today.

For future work, it is intended to use metaheuristics, to test this integer linear programming model in larger instances and/or instances from the literature, in an attempt to overcome other logistical difficulties in the home health care services.

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